

Supporting Learning 2  
Foundation Stage  
Key Stage 1  
Key Stage 2



## A guide to helping your child with calculations



How we teach addition, subtraction,  
multiplication and division.

**This booklet explains the different methods of calculation - and models and images – which are taught and used at Girton Glebe Primary School. It also details the order in which they are introduced. Our expectation is that all children will leave Girton Glebe with a “toolkit” of calculation methods from which they can choose the best, or most efficient, method.**

The following pages detail some of the key models and images which we use to expose the structure of maths, or the concept being taught. These are used alongside mental methods and more formal methods of calculation as a tool to expose mathematics and ensure a deeper understanding. This calculation policy also describes a combination of mental methods, methods which use a few jottings and formal methods. There is a strong understanding on understanding the maths behind these methods, which the models and images expose, as does ensuring that children have the opportunity to use them in real-life contexts and problems.

For each operation, we have identified the year group when the method will be taught and consolidated. The teaching of later methods does not mean that children stop using earlier ones; they simply expand their “toolkit”.

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## Models and images used to support mathematical understanding

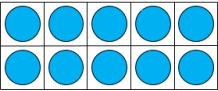
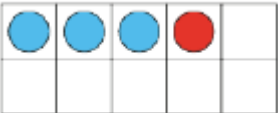

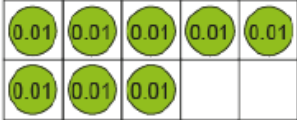
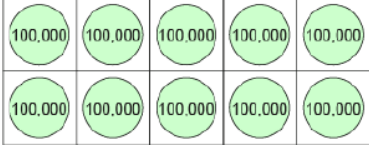
Across all year groups, and through all four operations, teachers and children are encouraged to use a range of models and images to support the children's development and understanding of both the mathematic concept being taught and the structure of the calculation.

Key resources that will be used include:

- Bead strings
- Multi-link cubes
- Numicon

Key models and images that will be used include:

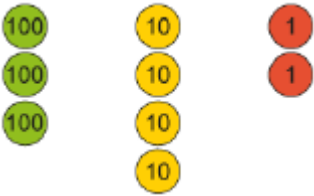
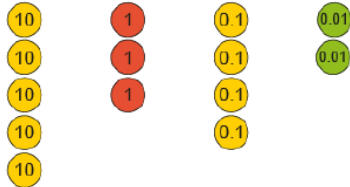
### Tens Frame

<u>EYFS/ Year 1 example</u>	<u>Year 4 example</u>	<u>Year 5-6 example</u>
 10   $3 + 1 = 4$	 1,000	  

**Sentence stem: There are 10 ones in 1 ten.**

**There are 10 hundreds in 1 thousand.**

### Place Value counters

<u>Year 3 example</u>	<u>Year 5/6 example</u>
	

Part-whole model

Reception example	Year 3 example	Year 5-6 example
<p>part 5 part 2 whole 7</p> <p>part 2 whole 5 part 3</p>	<p>342 40 300 2</p>	<p>53.42 0.4 3 0.02 50</p>

**Sentence stem: 5 is a part, 2 is a part, 7 is the whole.**

This is also an effective tool for teaching addition and subtraction and the inverse relationship between them both:

<p>Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.</p> <p>a)</p> <p>b)</p> <p><math>4 + 0 = 4</math> <math>3 + 1 = 4</math></p>	<p><i>I have completed this subtraction.</i> <math>525 - 270 = 255</math> <i>I will check using addition.</i></p> $  \begin{array}{r}  \text{H T O} \\  \hline  270 \\  + 255 \\  \hline  525 \\  \hline  \end{array}  $
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Place Value Grids

Year 4 example	Year 5-6 example

This is really effective for teaching column addition and subtraction, especially when exchanging.

**Sentence stem: exchange 10 ones for 1 ten. Etc**

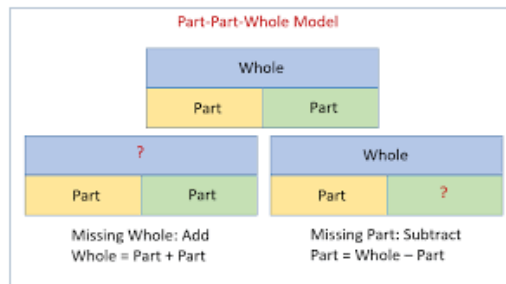
Diennes

Year 2 example	Year 3 example	Year 5 example
<p>Use known bonds and unitising to add 10s.</p> <p><i>I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.</i></p>	<p>Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.</p>	<p>Represent related facts with place value equipment when dividing by unitising.</p> <p><i>180 is 18 tens.</i></p> <p><i>18 tens divided into groups of 3 tens. There are 6 groups.</i></p> <p><i>180 ÷ 30 = 6</i></p>

Number line

Year 1 example	Year 4 example	Year 6 example

## Bar model



This can be used across year groups and across all strands of the maths curriculum:

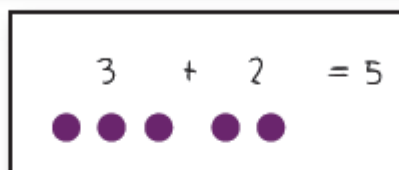
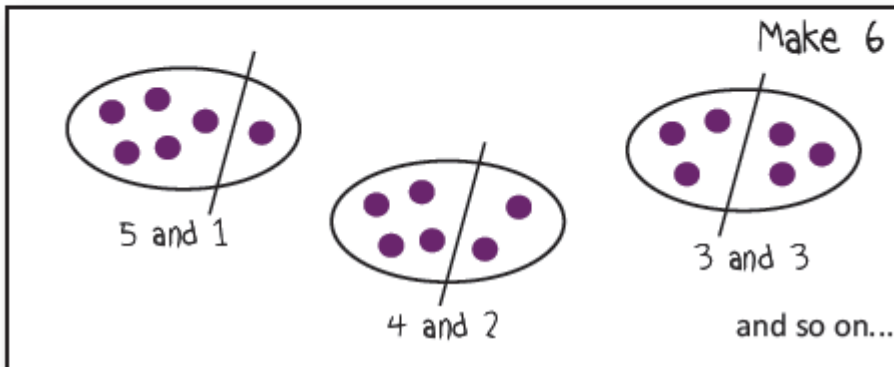
Addition and subtraction	Multiplication	Fractions																												
<p>Use bar models to represent subtractions where a part needs to be calculated.</p> <div style="text-align: center;"> <p>Total 5,762</p> <table border="1" style="margin: auto;"> <tr> <td style="width: 50px; height: 20px;">?</td> <td style="width: 50px; height: 20px;">2,899</td> </tr> <tr> <td style="text-align: center; font-size: small;">Yes votes</td> <td style="text-align: center; font-size: small;">No votes</td> </tr> </table> </div> <p><i>I can work out the total number of Yes votes using <math>5,762 - 2,899</math>.</i></p> <p>Bar models can also represent 'find the difference' as a subtraction problem.</p> <div style="margin-top: 20px;"> <table style="border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">Danny</td> <td style="border: 1px solid black; padding: 2px 10px; text-align: center;">899</td> <td style="padding: 0 10px;">← ? →</td> </tr> <tr> <td>Luis</td> <td style="border: 1px solid black; padding: 2px 10px; text-align: center;">1,005</td> <td></td> </tr> </table> </div>	?	2,899	Yes votes	No votes	Danny	899	← ? →	Luis	1,005		<p>A bar model may represent multiplications as equal groups.</p> <div style="text-align: center; margin: 10px 0;"> <table border="1" style="margin: auto;"> <tr> <td colspan="6" style="text-align: center; font-size: small;">24</td> </tr> <tr> <td style="width: 20px; height: 20px; text-align: center;">4</td> <td style="width: 20px; height: 20px; text-align: center;">4</td> <td style="width: 20px; height: 20px; text-align: center;">4</td> <td style="width: 20px; height: 20px; text-align: center;">4</td> <td style="width: 20px; height: 20px; text-align: center;">4</td> <td style="width: 20px; height: 20px; text-align: center;">4</td> </tr> </table> </div> <p><math>6 \times 4 = 24</math></p>	24						4	4	4	4	4	4	<p style="text-align: center;">What is <math>\frac{1}{3}</math> of 18?</p> <table border="1" style="margin: 10px auto; text-align: center;"> <tr> <td colspan="3" style="padding: 2px 10px;">18</td> </tr> <tr> <td style="width: 30px; height: 20px; background-color: yellow;">6</td> <td style="width: 30px; height: 20px;">6</td> <td style="width: 30px; height: 20px;">6</td> </tr> </table>	18			6	6	6
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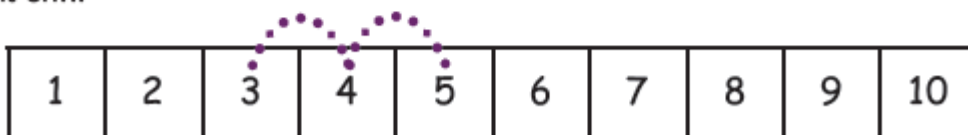
# Addition

## EYFS – Year 1

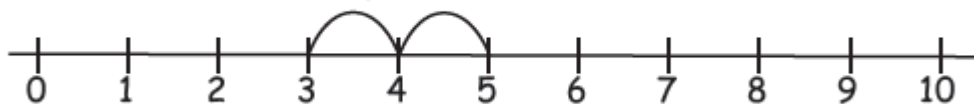
Children first use a range of objects and pictures to explain addition as combining two sets of objects. Beginning with the objects themselves, children learn to do addition by drawing pictures of the objects and later representing the objects by symbols such as dots or tally marks.



Once this process is secure, children begin to use a **number track** to help them count on...



...or a **numberline** marked in steps of 1.



Note: Labelled numberlines like this should always start at 0 (zero).

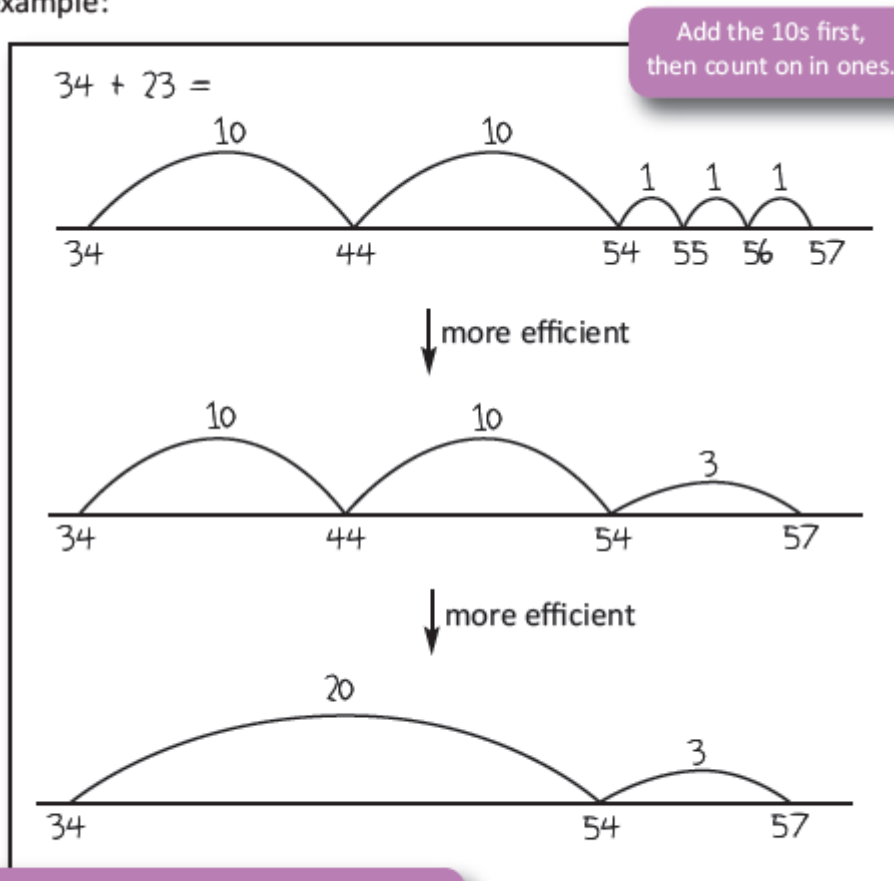
During Year 1 children will also use **hundred squares** and **bead strings** to help them calculate with numbers up to 20.

The use of **tens frames** are also pivotal during this.

## Year 2

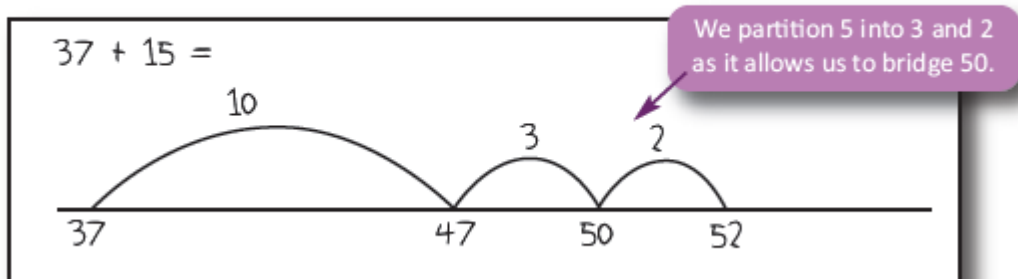
Empty number lines are used in Year 2 to count up. They will become increasingly efficient as children become more confident in choosing the jumps that they use.

For example:



Note: We don't use arrows on a number line so that children can see that counting on and counting back are the same process.

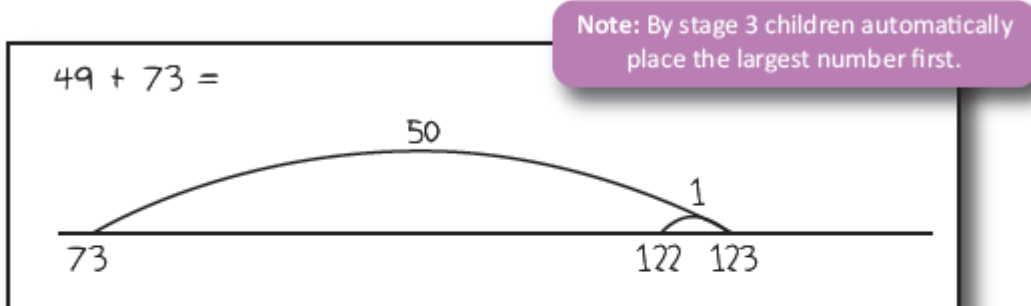
**Bridging** across 10 is another way of making the process more efficient.



Children will also use hundred squares, bead strings, place value counters and diennes to support their addition of numbers to 100. Part-whole models will be used alongside this.

## Year 2 -3

Children use the numberline to support addition for increasingly large numbers. In addition, they learn to add numbers such as 49 by adding 50 and taking away 1.



Pencil and paper methods without a number line begin at this stage by **partitioning**. For example:

$$\begin{aligned} 67 + 24 \\ = (60 + 20) + (7 + 4) \\ = 80 + 11 \\ = 91 \end{aligned}$$

Partitioning splits each number into the tens and units - we always add the tens first, then the units, as this is how we would do the calculation mentally.

## End of Year 3

Our first “written method” is an expanded version. Because this is a written method and not a mental one, and so that children are ready to “carry”, we begin from the right with the **least significant digits**.

$$\begin{array}{r} 67 + 24 = \\ E = 70 + 20 = 90 \end{array}$$

We make an estimate first: in this case by rounding the calculation to the nearest 10.

$$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \quad (7 + 4) \\ 80 \quad (60 + 20) \\ \hline 91 \end{array}$$

Add the units, 7 & 4, first. Then add the tens, 60 & 20; treating the tens in this way keeps the “sense” of the calculation.

Note: It is important that children have a good understanding of the calculation they are doing. They should make an estimate of the calculation first - in this way they should be able to spot any calculation errors. This is helped by making sure that calculations have a context in which they make sense.

## Year 3 – 6

The common standard written method of addition is now used. Children can be encouraged to make estimates to check the accuracy of their answer.

6 4 2 7 + 3 6 8 =  
E = 6 4 3 0 + 3 7 0 = 6 8 0 0

6 4 2 7  
+ 3 6 8  
-----  
5  
|

Here, 7 and 8 is 15, which is 5 in the units column and 1 ten carried into the tens column.

Then, 20 and 60 and the one ten is 90 which is 9 in the tens column. There's nothing to carry so we have 7 hundreds and 6 thousands.

6 4 2 7 + 3 6 8 =  
E = 6 4 3 0 + 3 7 0 = 6 8 0 0

6 4 2 7  
+ 3 6 8  
-----  
6 7 9 5      A = 6 7 9 5  
|

This standard written method should be used only when an easier or quicker method is not available. As children become more confident they will be able to use the method to:

- add more than two numbers with different numbers of digits
- add money, lining up the decimal points, and dealing with mixed amounts, eg £3.59 + 78p
- add two or more decimal fractions with up to two decimal places
- add quantities in mixed units, eg 3.2m + 280cm

Billy uses a pedometer to measure how far he walks in three days. On Monday he walked 4.6km, on Tuesday he walked 5km and on Wednesday he walked 780m. How far did he walk altogether?

4.6 km + 5 km + 780 m =  
E = 5 km + 5 km + 1 km = 11 km

4.6 0  
5.0 0  
0.7 8  
-----  
10.3 8      A = 10.3 8 km  
|

Children will also use practical resources and diagrams to help them add improper fractions and mixed numbers.



# Subtraction

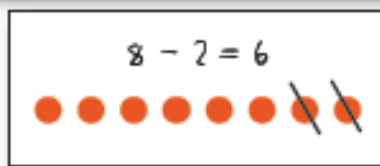
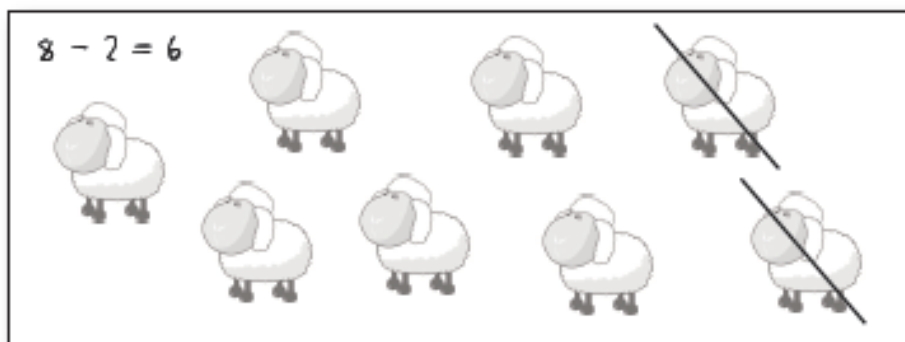
Subtraction can be described in three ways:

- taking away
- counting back
- finding the **difference** (counting on)

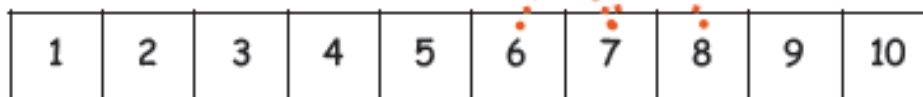
It is important that children understand the relationship between these three different interpretations of subtraction.

## Stage 1 (Foundation Stage - Year 1)

Real objects, pictures and symbols come first. Taking away is easiest.



Use a number track to count back:



Or find the difference by comparing two number tracks:



Note: Counting on with a numberline works too, but this way you can see a difference between the two numbers.

In Year 1, children will use number lines, bead strings and hundred squares to calculate numbers up to 20.

Children understand 'find the difference' as subtraction.

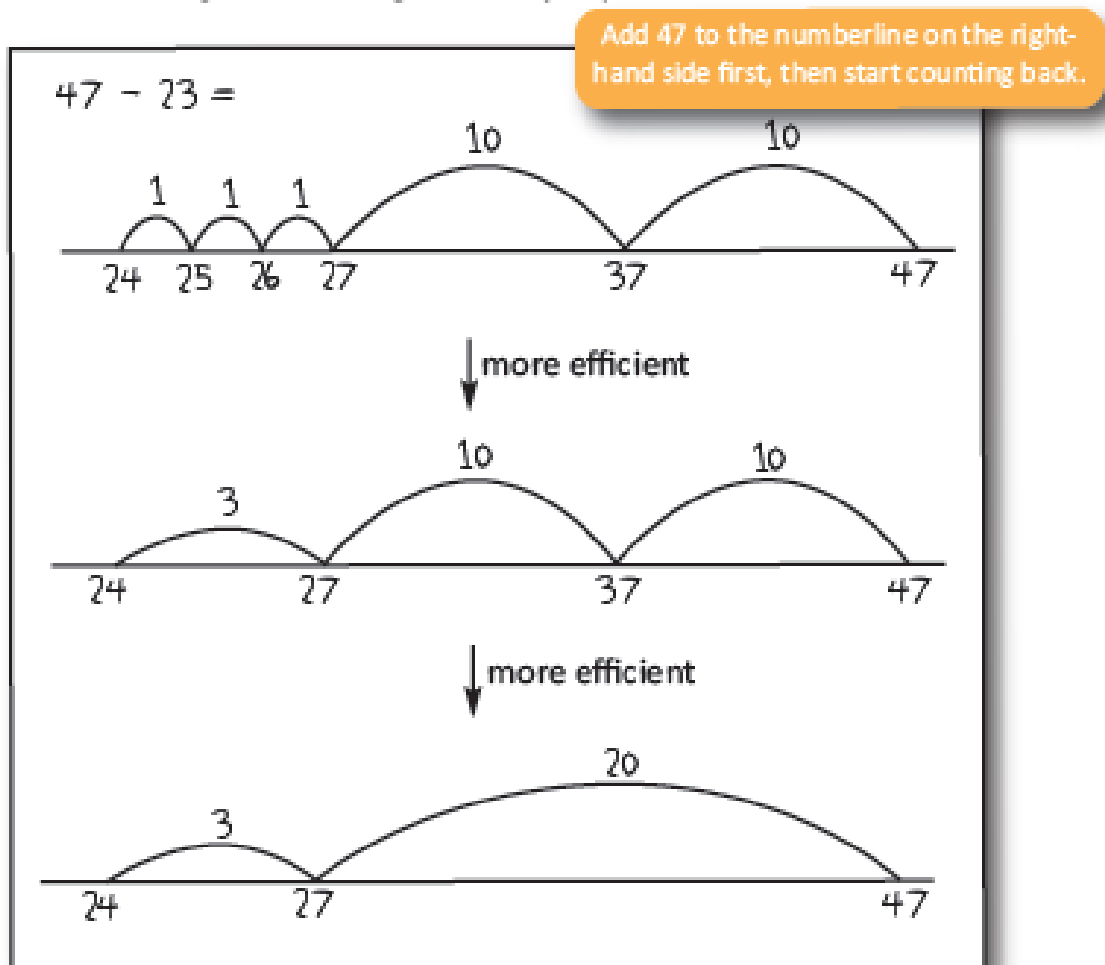


10 - 4 = 6

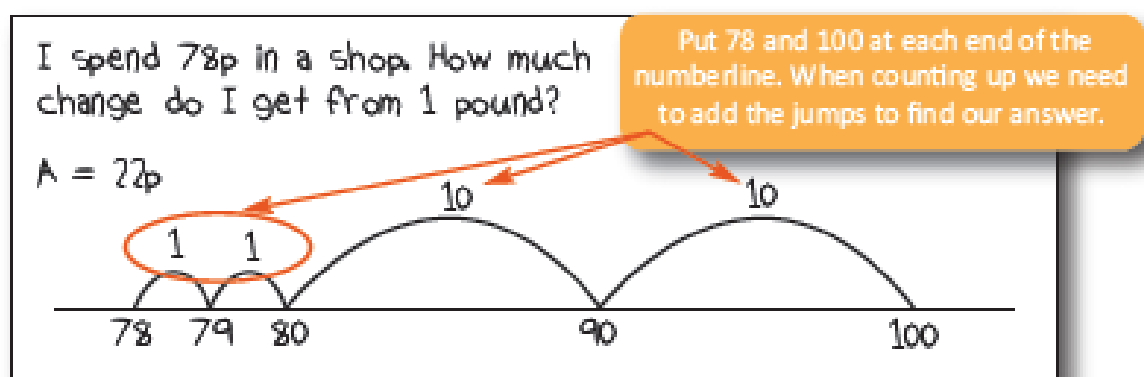
The difference between 10 and 6 is 4.

## Year 2

Using an empty numberline we count backwards from the right. As with addition, the challenge is choosing efficient jumps backwards.



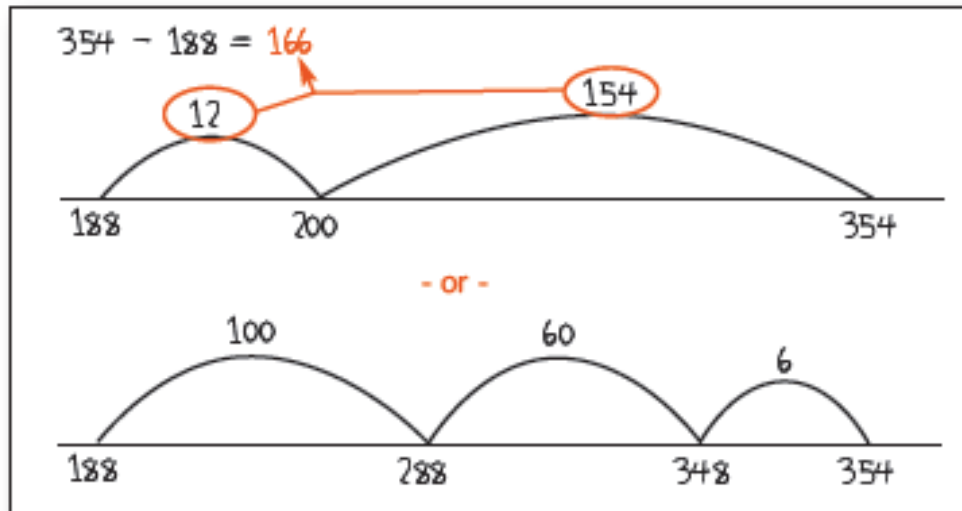
If the numbers involved in the calculation are close together or close to multiples of 10 or 100 etc, it can be easier to count on. In fact, many children find counting on more straightforward. A common example is counting on to find change.



Children will continue to use beadstrings and hundred squares to support their subtraction of numbers to 100.

## Beginning of Year 3

At this stage, children are using numberlines with much greater skill; they choose the best methods and the best jumps for subtraction calculations less than 1000.



Once children are efficient subtracting along a number line, they can be introduced to column subtraction. This makes it explicit to move tens into the ones column to carry out a subtraction such as  $63 - 37$ . As with addition, we **must start at the right** with the **least significant digits**.

	H	T	O
	1	<del>6</del>	15
-		3	8
	1	3	7

$$175 - 38 = 137$$

When exchanging, use the sentence stem "Exchange one ten for ten ones" whilst you show your working out.

Then, use the sentence stem "15 tens subtract 8 tens is 7 tens." Use this sentence stem with the tens, hundreds etc.

## Year 4 – 6

Build on the method of column subtraction introduced in Year 3.

$$754 - 286 =$$

$$E = 750 - 300 = 450$$

$$\begin{array}{r} 754 \\ - 286 \\ \hline \end{array}$$

In this calculation, first move a ten into the units column. A common mistake is to reverse the calculation, taking 4 from 6. Repeat, moving a hundred into the tens column.

$$754 - 286 =$$

$$E = 750 - 300 = 450$$

$$\begin{array}{r} 7\overset{4}{\cancel{5}}4 \\ - 286 \\ \hline 8 \end{array}$$

$$754 - 286 =$$

$$E = 750 - 300 = 450$$

$$\begin{array}{r} 6\overset{14}{\cancel{7}}\overset{14}{4} \\ - 286 \\ \hline 68 \end{array}$$

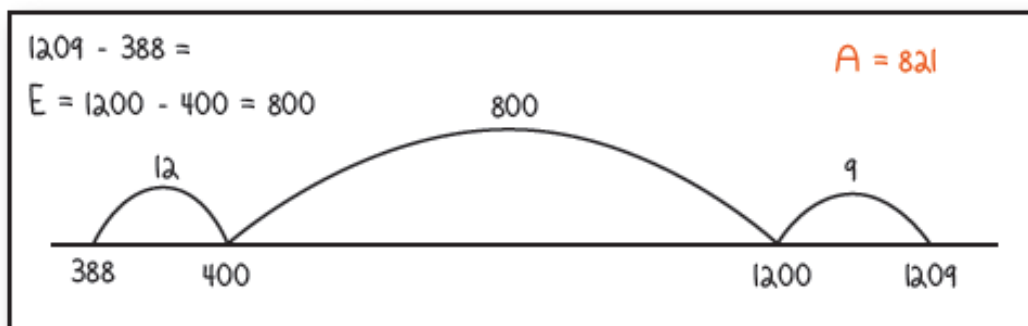
This method is highly error prone. It is essential that children check their answer against their estimate.

$$754 - 286 =$$

$$E = 750 - 300 = 450$$

$$\begin{array}{r} 6\overset{14}{\cancel{7}}\overset{14}{4} \\ - 286 \\ \hline 468 \end{array} \quad A = 468$$

Numberlines remain easier and more reliable in some cases and children by now should be confident with this process.



As with addition, children should be able to calculate with:

- decimal fractions with different numbers of digits
- different units, eg 5.67kg - 870g
- numbers with 5 or 6 digits (Year 5)
- improper fractions and mixed numbers



## Multiplication

### Stage 1 (Foundation Stage - Year 1)


Early work on multiplication involves counting on in steps of 2 initially, then in steps of 5 and 10. The concept of multiplication at this stage is **entirely** practical - it involves exploring real-life examples of equal sets or groups.

I have 4 pairs of socks. How many socks are there?



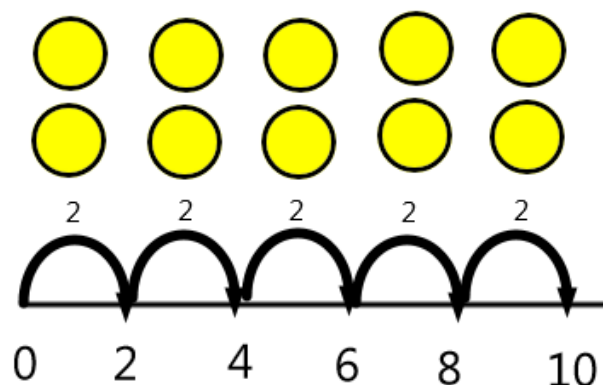
Just as with addition and subtraction, children can begin to substitute symbols for real objects.

I have 3 boxes of 6 eggs. How many eggs?



Representing numbers in this way, i.e. in a grid, is called an array. In this example you can also see that the array shows that 6 is 3 lots of 2 and also 2 lots of 3.

In year 1, using arrays alongside number line can really support the development and understanding of this method.



## Year 2

Here, we begin to represent multiplication as repeated addition. So the following expressions all show the same calculation:

3 times 5

$5 + 5 + 5$

3 equal groups 5

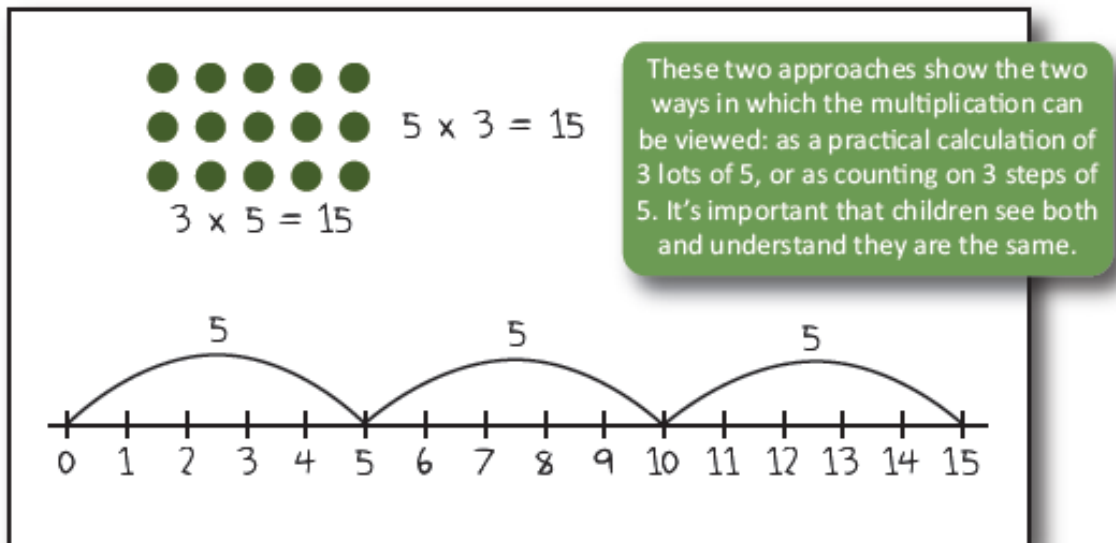
$3 \times 5$

Multiplication (like addition) is commutative: that is,  $3 \times 5$  is the same as  $5 \times 3$ . Children use this fact, with repeated addition, to calculate simple multiplications.

$4 \times 7$ $= 7 + 7 + 7 + 7$ $= 28$	$7 \times 4$ $= 4 + 4 + 4 + 4 + 4 + 4 + 4$ $= 28$
---	---

## Year 2 and beginning of Year 3

Using a grid (array) or a numberline, we can calculate a multiplication:



Both of these methods are used and are taught alongside each other and the relevant multiplication tables for the relevant year groups:

- 2,5 and 10 (Year 2)
- 3,4 and 8 (Year 3)
- All, up to and including,  $12 \times 12$  (Year 4)

### Year 3

This stage introduces 'grid method'.

$$\begin{array}{l} 23 \times 8 = \\ E = 25 \times 8 = 200 \\ \begin{array}{r|l|l} \times & 20 & 3 \\ \hline 8 & & \end{array} \end{array}$$

We complete an estimate first so that we can check our answer. Then we partition the two-digit number into its tens (20) and units (3). Set the question out in a grid as shown.

Now calculate  $8 \times 20$  and place the answer in the grid, following this with  $8 \times 3$ . Add the two answers together to complete the calculation and check with the estimate.

$$\begin{array}{l} 23 \times 8 = 184 \\ E = 25 \times 8 = 200 \\ \begin{array}{r|l|l} \times & 20 & 3 \\ \hline 8 & 160 & 24 \end{array} \quad \begin{array}{r} 160 \\ + 24 \\ \hline 184 \end{array} \end{array}$$

### Year 4

Here, we begin to use the formal column method. We understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.

$$\begin{array}{r} \phantom{2}3 \\ \times \phantom{2}5 \\ \hline \phantom{2}15 \\ 100 \\ \hline 115 \end{array}$$

$$\begin{array}{r} \phantom{2}3 \\ \times \phantom{2}5 \\ \hline 115 \\ \phantom{11}1 \\ \hline \phantom{11}1 \end{array}$$

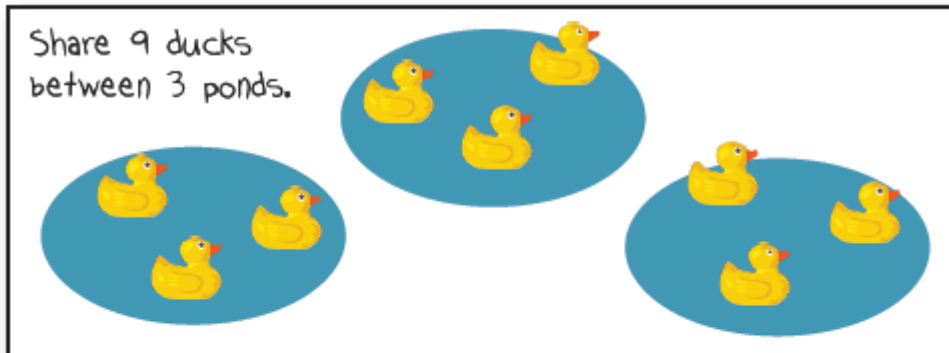




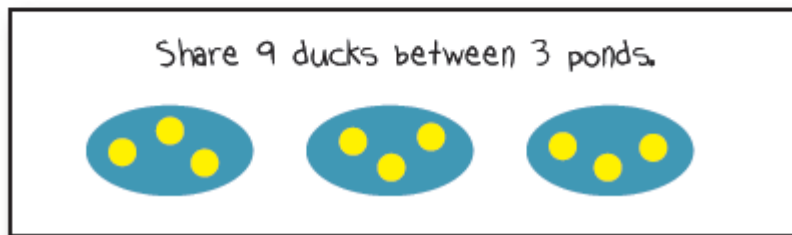
## Division

### Foundation Stage and Year 1

Early division involves making equal groups from a whole in practical and real-life contexts.

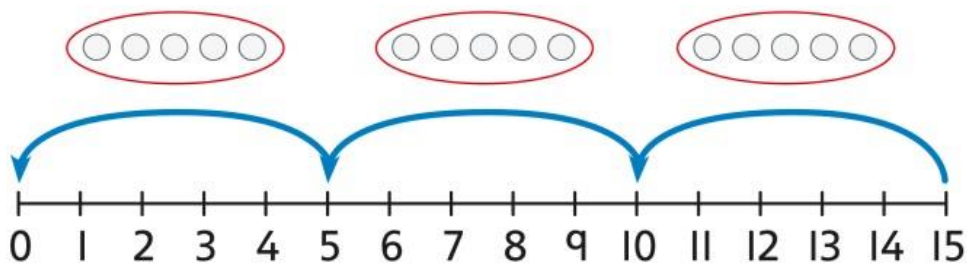


The same problem can be represented with symbols:



As we move into Year 1, this can be represented on a number line.

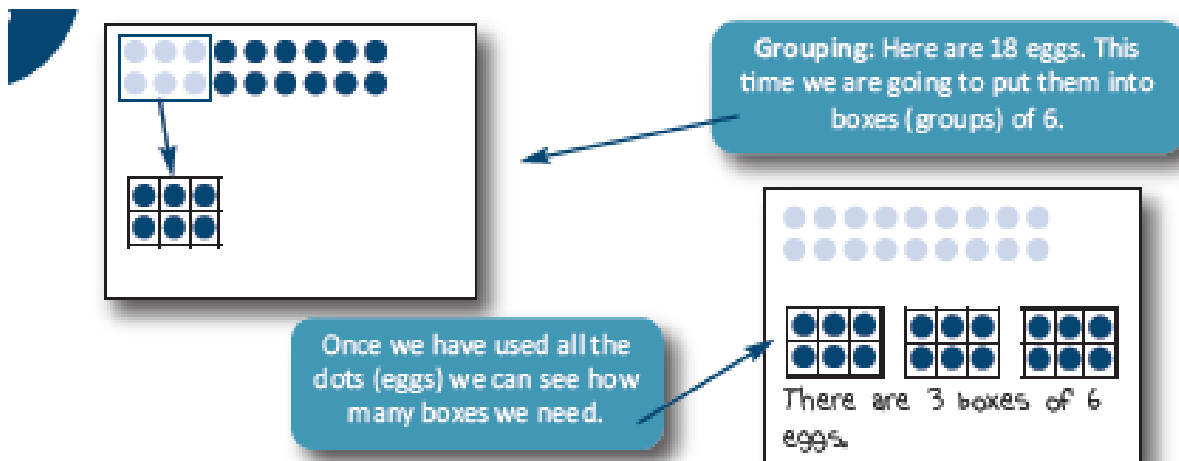
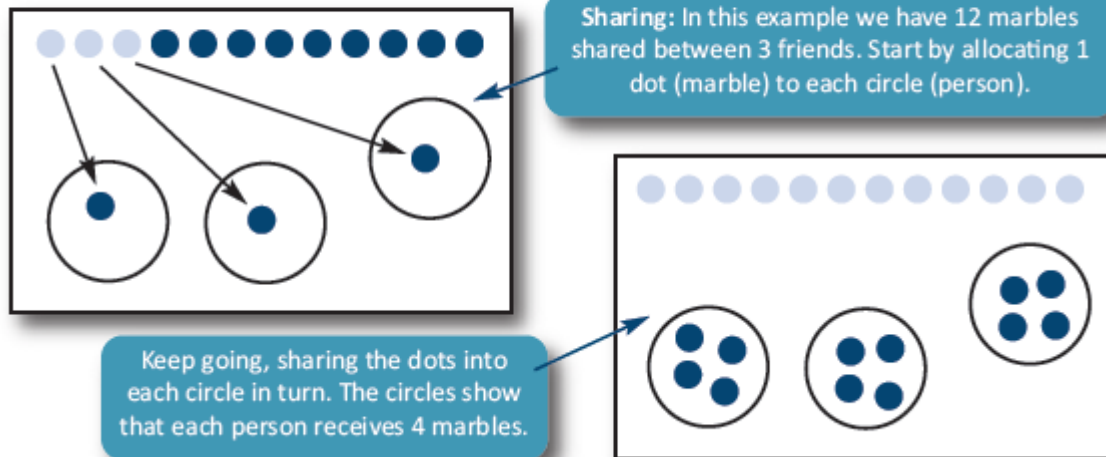
Grouping to represent  $15 \div 5 = 3$



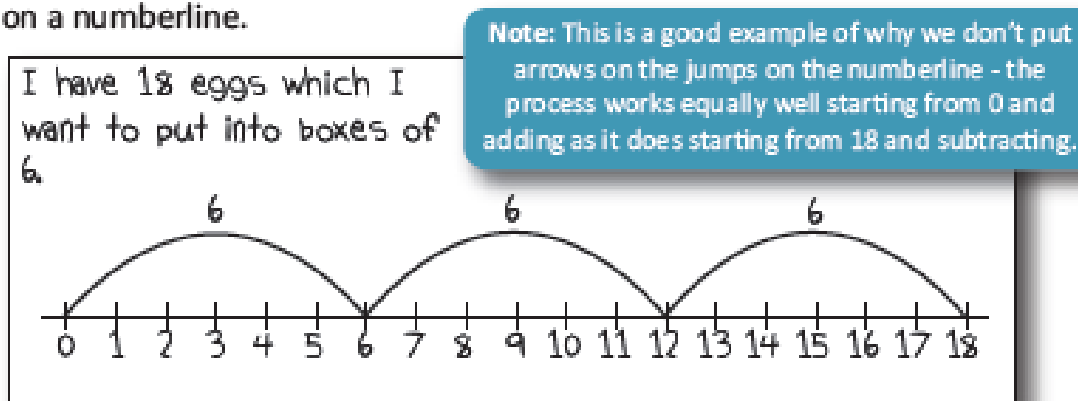
## Year 2

At stage 2, children develop their understanding of division as two separate processes:

- sharing (eg 12 marbles shared between 3 friends)
- grouping (eg 18 eggs are put into boxes of 6)



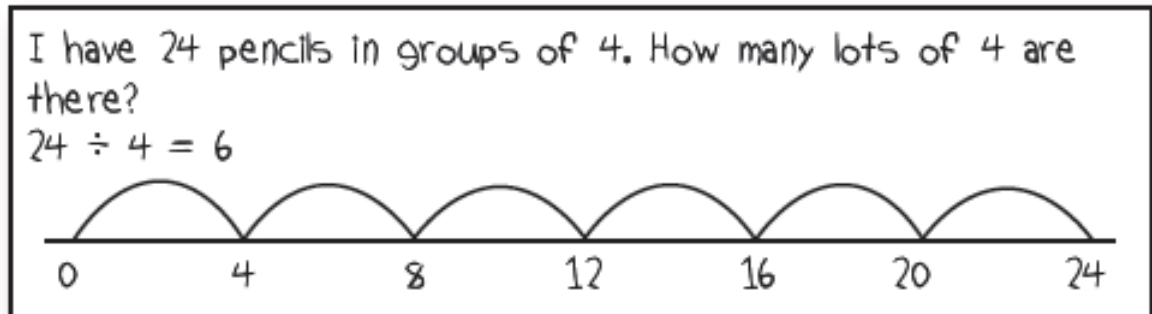
In the same way we can use repeated addition to show the same process, that is, we repeatedly add groups of 6 until we can't any longer. It is possible to show this on a numberline.



At this stage, we do not deal with remainders.

### Year 3

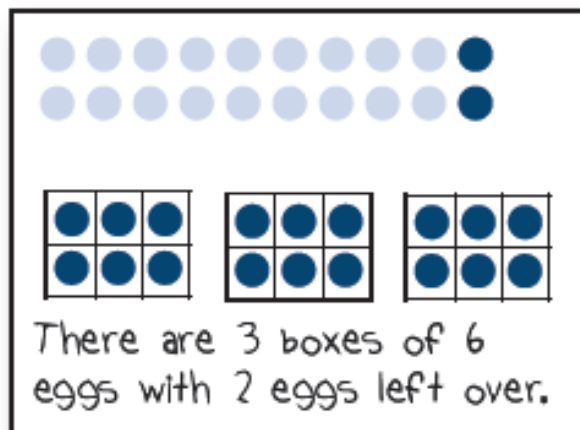
Firstly, children can carry out repeated addition on a blank numberline for a calculation with no remainder.



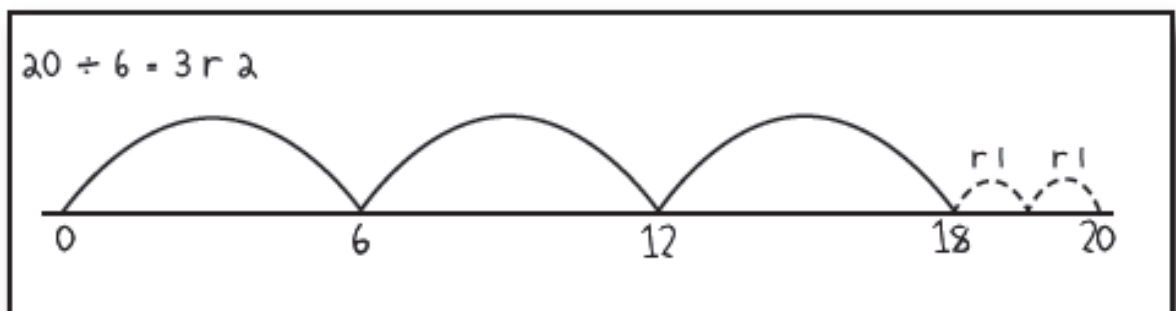
Children will use known multiplication facts to help with division.

I know  $4 \times 6 = 24$  so  $24 \div 4 = 6$

Then they can see the effect of having a remainder. So, repeating the earlier example of putting eggs in boxes but this time with 20 eggs:

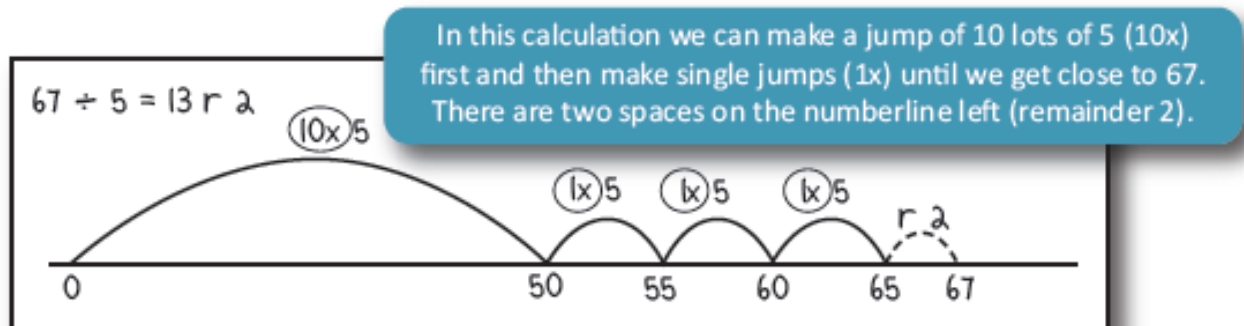


Or on a numberline:

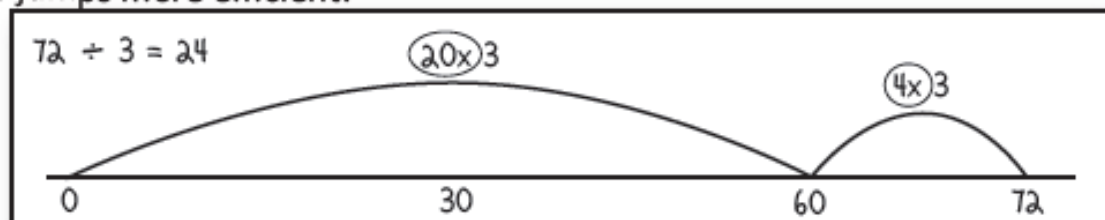


## Year 4

In Year 4, we start to make this process more efficient by grouping some individual steps into one 'chunk'; for example:



By grouping more than one step as the numbers get larger we can make several larger jumps to the target number. Related multiplication facts are useful to make jumps more efficient.



At this stage, children also divide whole numbers by 10 and 100, extending their knowledge of place value including decimals.

## Year 5

At this stage children will multiply and divide whole numbers and decimals by 10, 100 and 1000, drawing on known multiplication facts.

Children will continue to use division on a number line for some questions.

**Short Division:** children will begin to use formal written methods for three-digit (HTU) numbers divided by single-digit (U) numbers.

$$291 \div 3 =$$

$$E = 100$$

$$3 \overline{) 291}$$

First calculate the number of 3s in 29 - in reality this is the number of threes in 290.

$$291 \div 3 =$$

$$E = 100$$

$$3 \overline{) 291}$$

There are 9 threes, making 27...

$$291 \div 3 =$$

$$E = 100$$

$$3 \overline{) 291}$$

...with 2 left over which is carried into the next column as tens.

$$291 \div 3 =$$

$$E = 100$$

$$3 \overline{) 291} \begin{array}{l} 97 \\ \hline \end{array}$$

$$A = 97$$

Finally, calculate the number of threes in 21.

Children should be able to interpret the remainder as a fraction or decimal, for example:

$$432 \div 5 =$$

$$E 400 \div 5 = 80$$

$$5 \overline{) 432} \begin{array}{l} 86r2 \\ \hline \end{array}$$

$$A = 86r2$$

$$= 86 \frac{2}{5}$$

$$432 \div 5 =$$

$$E 400 \div 5 = 80$$

$$5 \overline{) 432.20} \begin{array}{l} 86.4 \\ \hline \end{array}$$

$$A = 86.4$$

## Year 6

Finally, long division allows us to tackle calculations where we want to divide by a two-digit number.

Children will continue to use division on a number line for some questions.

$$563 \div 24 =$$

$$E = 600 \div 25 = 24$$

$$\begin{array}{r} 24 \overline{) 563} \\ \underline{- 480} \\ 83 \end{array}$$

Start by finding the number of 24s in 56 (we know there are no 24s in 5). 2 x 24 is 48, so our first lot is 480. We can therefore put 2 in the tens column at the top (our answer) and subtract our chunk, leaving 83 to do next.

Next we look for the number of 24s in 83. As 3 x 24 is 72, we can put these 3 lots of 24 into our answer and again subtract the lot of 72. With 11 remaining, there are no more 24s available.

$$563 \div 24 =$$

$$E = 600 \div 25 = 24$$

$$\begin{array}{r} 24 \overline{) 563} \\ \underline{- 480} \\ 83 \\ \underline{- 72} \\ 11 \end{array}$$

$A = 23 \text{ r } 11$

Children should be able to interpret the remainder as a fraction or decimal, for example:

$$432 \div 15 =$$

$$E = 450 \div 15 = 30$$

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{300} \quad (15 \times 20) \\ 132 \\ \underline{120} \quad (15 \times 8) \\ 12 \end{array}$$

$\frac{12}{15} = \frac{4}{5} \quad A = 28 \frac{4}{5}$

$$432 \div 15 =$$

$$E = 450 \div 15 = 30$$

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \quad \downarrow \\ 132 \quad \downarrow \\ \underline{120} \quad \downarrow \\ 120 \\ \underline{120} \\ 0 \end{array}$$

$A = 28.8$

**array** - an organised collection of objects, counters or symbols, for example arranged in rows and columns



**bridging** - the process of using a multiple of 10 or 100 as part of an addition or subtraction calculation, for example  $45 + 13$  can be thought of as  $45 + 5 (50) + 8$

**decomposition** - the standard written method for subtraction (see p10)

**difference** - the amount by which one number is greater than another - i.e. the result of a subtraction; the difference between 5 and 9 is 4

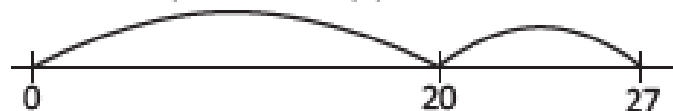
**grid method** - a method of calculating multiplication by separating the calculation into sections, each of which easier than the whole (see p12)

**least significant digits** - the digits with least value - usually the units

**number track** - a line of numbers used for counting or calculating, each section represents one number



**numberline** - a line where numbers are represented by points on it; numberlines always run from left to right



**partitioning** - separating a number into its different parts, eg 25 can be partitioned into 20 and 5

**remainder** - the amount left over in a division which cannot be grouped or shared equally

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**Girton Glebe Primary School**  
 Cambridge Road, Girton, Cambridge, CB3 0PN  
[www.girtonglebe.com](http://www.girtonglebe.com)  
 (01223) 276484      [office@girtonglebe.cambs.sch.uk](mailto:office@girtonglebe.cambs.sch.uk)