Supporting Learning 2

Foundation Stage

Key Stage 1

Key Stage 2

Girton Glebe School

A guide to

helping your child with calculations







How we teach addition, subtraction, multiplication and division.

This booklet explains the different methods of calculation which are taught at Girton Glebe Primary School and the order in which they are introduced. Our expectation is that children will leave primary school with a "toolkit" of calculation methods from which they can choose the best or most efficient method.

The following pages describe a combination of mental methods, methods which use a few jottings and formal written methods similar to those which many adults today learnt at school. There is a strong focus on *understanding* the maths behind these methods by ensuring that children have the opportunity to use them when solving real-life problems and challenges.

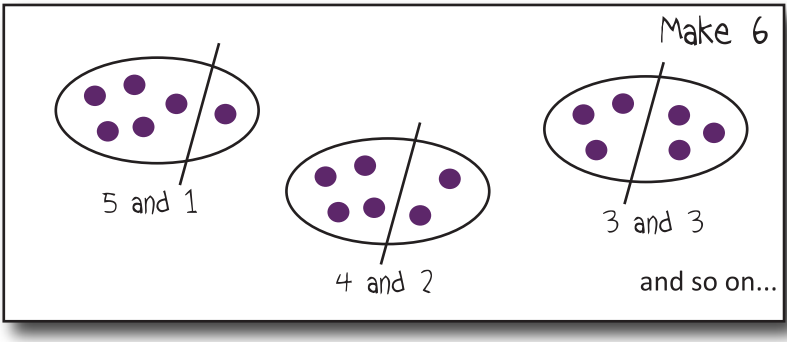
In each case we have identified the age range when the method will be taught and consolidated. The teaching of later methods does not mean that children stop using earlier ones; they simply expand their "toolkit".

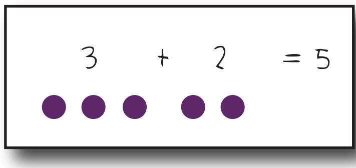
Contents

Addition

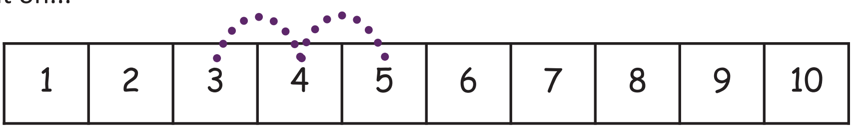
Stage 1 (Foundation Stage - Year 1)

Children first use a range of objects and pictures to explain addition as combining two sets of objects. Beginning with the objects themselves, children learn to do addition by drawing pictures of the objects and later representing the objects by symbols such as dots or tally marks.

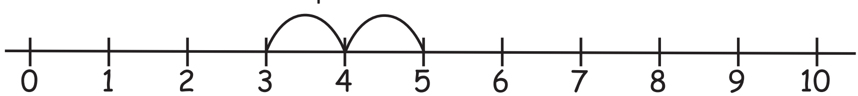




Once this process is secure, children begin to use a **number track** to help them count on...



...or a **numberline** marked in steps of 1.



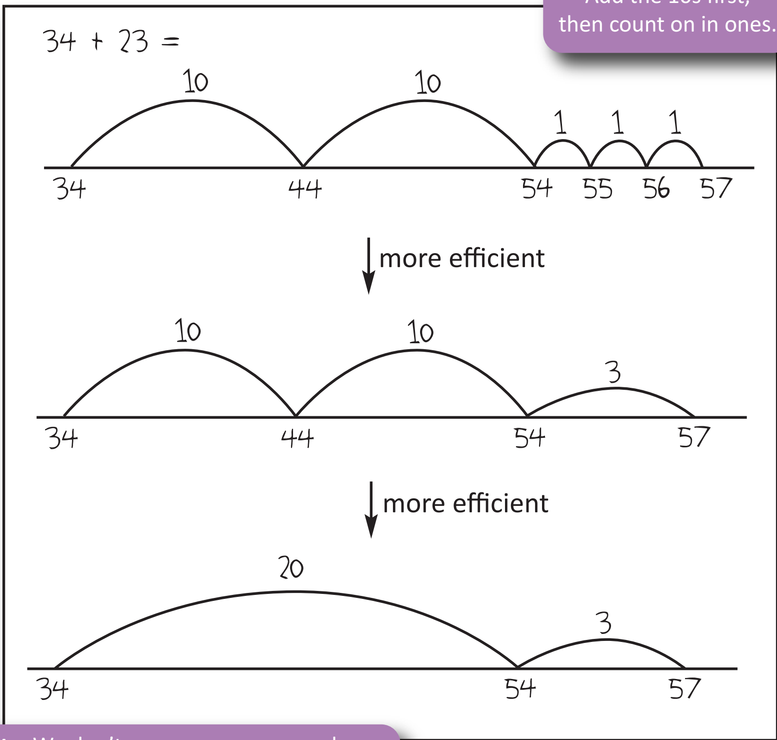
Note: Labelled numberlines like this should always start at 0 (zero).

During Year 1 children will also use **hundred squares** and **bead strings** to help them calculate with numbers up to 20.

Stage 2 (around Year 2)

Empty numberlines are used at stage 2 to count up, becoming increasingly efficient as children become more confident in choosing the jumps that they use. For example:

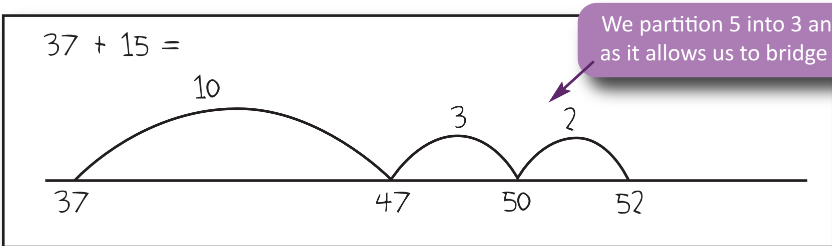
Add the 10s first, then count on in ones.



Note: We don't use arrows on a number line so that children can see that counting on and counting back are the same process.

Bridging across 10 is another way of making the process more efficient.

We partition 5 into 3 and 2 as it allows us to bridge 50.



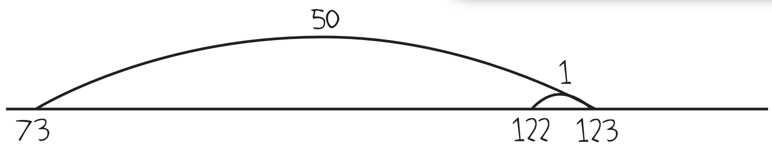
Children will also use **hundred squares** and **bead strings** to support their addition of numbers to 100.

Stage 3 (Year 2-3)

Children use the numberline to support addition for increasingly large numbers. In addition, they learn to add numbers such as 49 by adding 50 and taking away 1.

Note: By stage 3 children automatically place the largest number first.

49 + 73 =



Pencil and paper methods without a number line begin at this stage by **partitioning**. For example:

Partitioning splits each number into the tens and units - we always add the tens first, then the units, as this is how we would do the calculation mentally.

67 + 24
= (60 + 20) + (7 + 4)
= 80 + 11
= 91

Stage 4 (end Year 3 - Year 4)

Our first "written method" is an expanded version. Because this is a written method and not a mental one, and so that children are ready to "carry", we **begin from the right** with the **least significant digits**.

6 7 + 2 4 =
E = 7 0 + 2 0 = 9 0

We make an estimate first: in this case by rounding the calculation to the nearest 10.

6 7	
+ 2 4	
1 1	(7 + 4)
8 0	(6 0 + 2 0)
9 1	

Add the units, 7 & 4, first. Then add the tens, 60 & 20; treating the tens in this way keeps the "sense" of the calculation.

Note: It is important that children have a good understanding of the calculation they are doing. They should make an estimate of the calculation first - in this way they should be able to spot any calculation errors. This is helped by making sure that calculations have a context in which they make sense.

Stage 5 (Year 4 - Year 6)

The common standard written method for addition is used at stage 5, making sure that children have made an estimate first to pick up any errors.

6 4 2 7 + 3 6 8 =
E = 6 4 3 0 + 3 7 0 = 6 8 0 0

```
  6 4 2 7
+   3 6 8
---------
        5
---------
      1
```

Here, 7 and 8 is 15, which is 5 in the units column and 1 ten carried into the tens column.

Then, 20 and 60 and the one ten is 90 which is 9 in the tens column. There's nothing to carry so we have 7 hundreds and 6 thousands.

6 4 2 7 + 3 6 8 =
E = 6 4 3 0 + 3 7 0 = 6 8 0 0

```
  6 4 2 7
+   3 6 8
---------
  6 7 9 5      A = 6 7 9 5
---------
      1
```

This standard written method should be used only when an easier or quicker method is not available. As children become more confident they will be able to use the method to:

- add more than two numbers with different numbers of digits
- add money, lining up the decimal points, and dealing with mixed amounts, eg £3.59 + 78p
- add two or more decimal fractions with up to two decimal places
- add quantities in mixed units, eg 3.2m + 280cm

Billy uses a pedometer to measure how far he walks in three days. On Monday he walked 4.6km, on Tuesday he walked 5km and on Wednesday he walked 780m. How far did he walk altogether?

4.6 km + 5 km + 780 m =
E = 5 km + 5 km + 1 km = 11 km

```
   4.6 0
   5.0 0
   0.7 8
 -------
 1 0.3 8      A = 10.38 km
 -------
   1
```

Children will also use practical resources and diagrams to help them add improper fractions and mixed numbers.

Subtraction

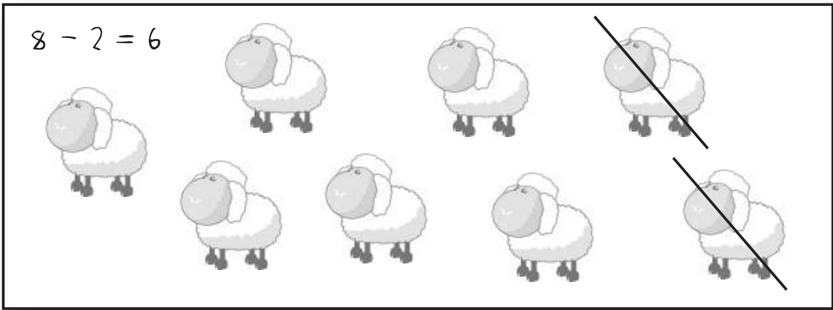
Subtraction can be described in three ways:

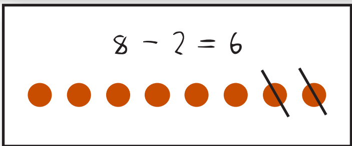
- taking away
- counting back
- finding the **difference** (counting on)

It is important that children understand the relationship between these three different interpretations of subtraction.

Stage 1 (Foundation Stage - Year 1)

Real objects, pictures and symbols come first. Taking away is easiest.





Use a number track to count back:

1	2	3	4	5	6	7	8	9	10

Or find the difference by comparing two number tracks:

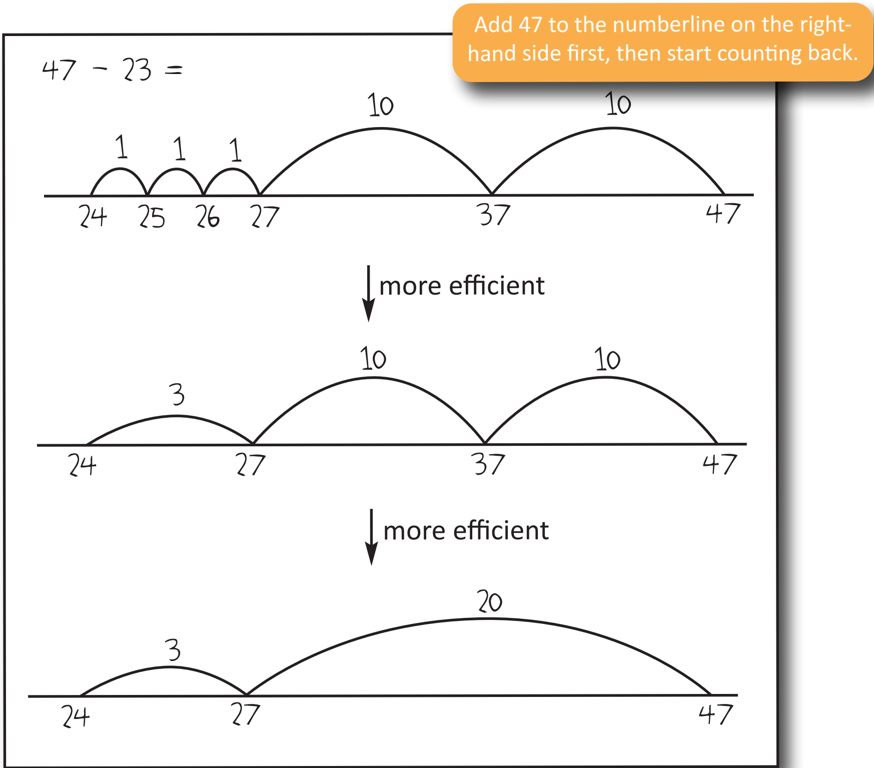
1	2	3	4	5	6		
1	2	3	4	5	6	7	8

Note: Counting on with a numberline works too, but this way you can see a **difference** between the two numbers.

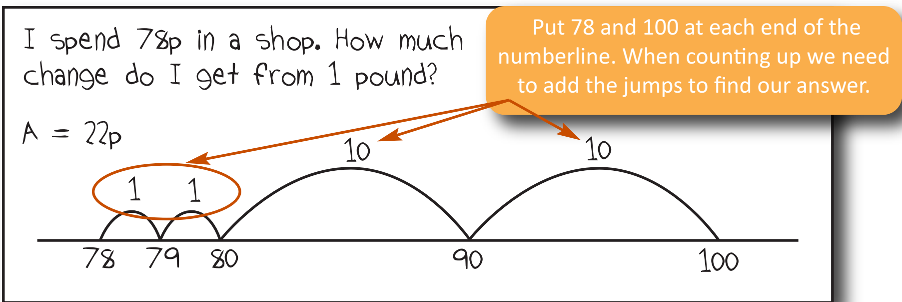
In Year 1, children will use number lines, bead strings and hundred squares to calculate numbers up to 20.

Stage 2 (around Year 2)

Using an empty numberline we count backwards from the right. As with addition, the challenge is choosing efficient jumps backwards.



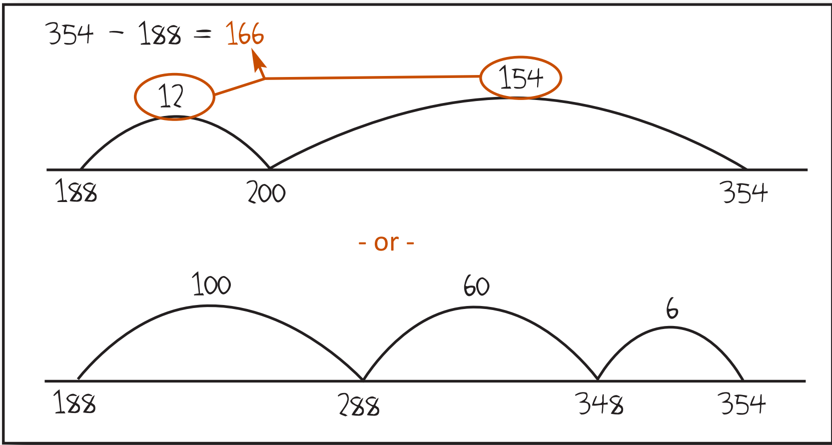
If the numbers involved in the calculation are close together or close to multiples of 10 or 100 etc, it can be easier to count on. In fact, many children find counting on more straightforward. A common example is counting on to find change.



Children will continue to use beadstrings and hundred squares to support their subtraction of numbers to 100.

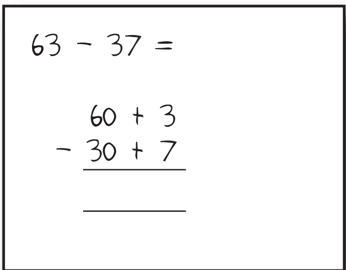
Stage 3 (around Year 3)

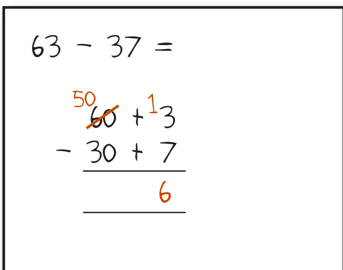
At this stage, children are using numberlines with much greater skill; they choose the best methods and the best jumps for subtraction calculations less than 1000.

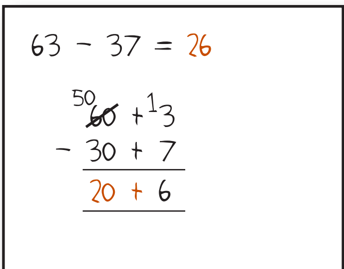


Pencil and paper methods without using a numberline begin with a simple, expanded method which makes explicit the need to move tens into the units column to carry out a subtraction like 63 - 37. As with addition, we must **start at the right** with the **least significant digits**.

Set out the calculation as shown here on the left, **partitioning** the tens and units. In order to "take 7 from 3" we need to move ten to the units column. This is called **exchanging**. At this point a common mistake is to reverse the calculation and take 3 from 7; remember that we are taking 37 from 63.







Once we have moved 10 we can take 7 from 13, leaving 6. Then we take 30 from the new value of 50.

Stage 4 (Year 4 - Year 6)

This is the final stage for subtraction. We contract our expanded written method into the standard method: **decomposition**. Again, start on the right.

754 - 286 =
E = 750 - 300 = 450

$$\begin{array}{r} 754 \\ -\ 286 \\ \hline \\ \hline \end{array}$$

In this calculation, first move a ten into the units column. A common mistake is to reverse the calculation, taking 4 from 6. Repeat, moving a hundred into the tens column.

754 - 286 =
E = 750 - 300 = 450

$$\begin{array}{r} 7\ {}^{4}\cancel{5}\ {}^{1}4 \\ -\ 2\ 8\ 6 \\ \hline 8 \\ \hline \end{array}$$

754 - 286 =
E = 750 - 300 = 450

$$\begin{array}{r} {}^{6}\cancel{7}\ {}^{14}\cancel{5}\ {}^{1}4 \\ -\ 2\ 8\ 6 \\ \hline 6\ 8 \\ \hline \end{array}$$

754 - 286 =
E = 750 - 300 = 450

$$\begin{array}{r} {}^{6}\cancel{7}\ {}^{14}\cancel{5}\ {}^{1}4 \\ -\ 2\ 8\ 6 \\ \hline 4\ 6\ 8 \\ \hline \end{array}$$

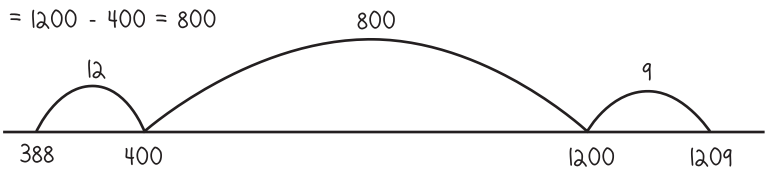
A = 468

This method is highly error prone. It is essential that children check their answer against their estimate.

Numberlines remain easier and more reliable in some cases and children by now should be confident with this process.

1209 - 388 =
E = 1200 - 400 = 800

A = 821



As with addition, children should be able to calculate with:

- decimal fractions with different numbers of digits
- different units, eg 5.67kg - 870g
- numbers with 5 or 6 digits (Year 5)
- improper fractions and mixed numbers

Multiplication

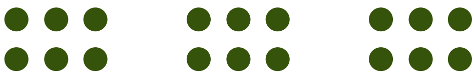
Stage 1 (Foundation Stage - Year 1)

Early work on multiplication involves counting on in steps of 2 initially, then in steps of 5 and 10. The concept of multiplication at this stage is **entirely** practical - it involves exploring real-life examples of equal sets or groups.

I have 4 pairs of socks. How many socks are there?

Just as with addition and subtraction, children can begin to substitute symbols for real objects.

I have 3 boxes of 6 eggs. How many eggs?



Representing numbers in this way, i.e. in a grid, is called an **array**. In this example you can also see that the array shows that 6 is 3 lots of 2 and also 2 lots of 3.

Stage 2/3 (Year 2)

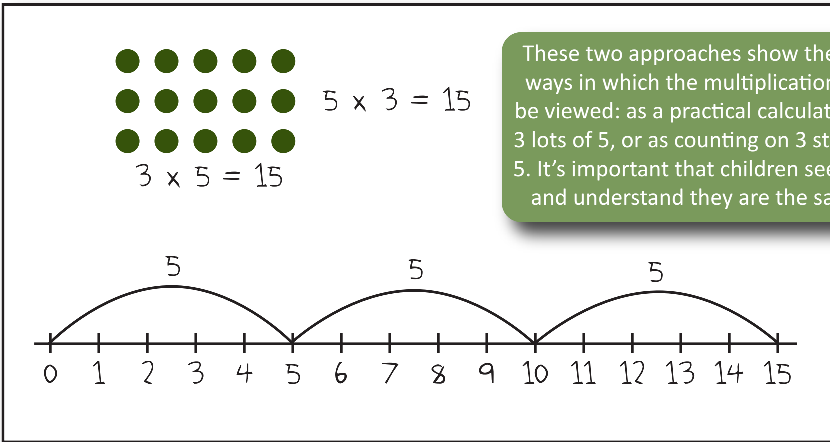
At stages 2 and 3 we represent multiplication as repeated addition. So, the following expressions all show the same calculation:

3 times 5   5 + 5 + 5   3 lots of 5   3 x 5

Multiplication (like addition) is commutative: that is, 3 x 5 is the same as 5 x 3. Children use this fact, with repeated addition, to calculate simple multiplications.

4 x 7	7 x 4
= 7 + 7 + 7 + 7	= 4 + 4 + 4 + 4 + 4 + 4 + 4
=28	= 28

Using a grid (**array**) or a numberline, we can calculate a multiplication:



These two approaches show the two ways in which the multiplication can be viewed: as a practical calculation of 3 lots of 5, or as counting on 3 steps of 5. It's important that children see both and understand they are the same.

Both of these methods are used throughout stages 2 and 3 and are taught alongside the relevant tables in the following order:

- 2, 5 & 10 times tables (Year 2)
- 3, 4 & 8 times tables (Year 3)
- 6, 7, 9, 11 & 12 times tables (Year 4)

Stage 4 (Year 3 and 4)

This stage introduces the '**grid method**' for multiplication. We begin with a straightforward calculation with a two-digit number (TU) multiplied by a single-digit number (U). Children will also use the grid method for three-digit numbers (HTU) multiplied by single-digit (U) numbers.

23 X 8 =
E = 25 X 8 = 200

X	20	3
8		

We complete an estimate first so that we can check our answer. Then we partition the two-digit number into its tens (20) and units (3). Set the question out in a grid as shown.

Now calculate 8 x 20 and place the answer in the grid, following this with 8 x 3. Add the two answers together the complete the calculation and check with the estimate.

23 X 8 = 184
E = 25 X 8 = 200

X	20	3
8	160	24

$$\begin{array}{r} 160 \\ +\ 24 \\ \hline 184 \end{array}$$

By the end of stage 4, children will be able to use a formal written method of calculation for two-digit (TU) and three-digit (HTU) multiplied by a single digit. This is taught alongside the grid method which most children find easier to understand.

350 x 7 = 2450

```
   3 5 0
x      7
--------
 2 4 5 0
--------
 2 3
```

Stage 5 (Year 5 - Year 6)

Stage 5 builds on stage 4 by extending the grid method to a range of other possible calculations.

- ThHTU x U (eg 4346 x 8)
- TU x TU (eg 72 x 38) & HTU x TU (eg 372 x 24 - example 1 below)
- U.t x U (eg 4.9 x 3) & U.th x U (eg 6.73 x 7 - example 2 below)

In **example 1** there are two rows in the grid - one for the tens and one for the units.

372 X 24 =

E = 400 X 25 = 10,000

X	300	70	2
20	6000	1400	40
4	1200	280	8

```
   6 0 0 0
   1 2 0 0
   1 4 0 0
     2 8 0
       4 0
+        8
----------
   8 9 2 8
----------
     1
```

A = 8,928

6.73 X 7 =

E = 7 X 7 = 49

Example 2 requires a good understanding of decimals.

X	6	0.7	0.03
7	42	4.9	0.21

```
   4 2
    4.9
+   0.2 1
---------
  4 7.1 1
---------
    1
```

A = 47.11

The final stage for this operation is the standard written method of long multiplication. It is easy to see how this method develops from the grid method as the processes are the same, with each section of the grid written in a column.

56 x 27 =

E = 60 x 25 = 1500

$$\begin{array}{rr} & 56 \\ \times & 27 \\ \hline \end{array}$$

Remembering to estimate first, set out the calculation lining up the tens and units. There are four calculations: 50x20, 6x20, 50x7 & 6x7. Write each of these on a separate line.

56 x 27 =

E = 60 x 25 = 1500

$$\begin{array}{rrl} & 56 & \\ \times & 27 & \\ \hline & 1000 & (50 \times 20) \\ & 120 & (6 \times 20) \\ & 350 & (50 \times 7) \\ & 42 & (6 \times 7) \\ \hline & 1512 & \\ \hline & 1 & \end{array}$$

Now the simple process of totalling the four lines is all that is left to do. Then check the answer against the estimate.

$$\begin{array}{rrl} & 56 & \\ \times & 27 & \\ \hline & 1120 & (56 \times 20) \\ & 392 & (56 \times 7) \\ \hline & 1512 & \\ \hline \end{array}$$

Once this method is understood, it can be further shortened and the four additions replaced by two

Even at this stage, many children prefer the visual nature of the grid method.

A school trip to Wimpole Hall costs £4.63 for each child. 23 children go on the trip. How much does it cost altogether?

E = £5 x 23 = £115

x	4	0.6	0.03
20	80	12	0.6
3	12	1.8	0.09

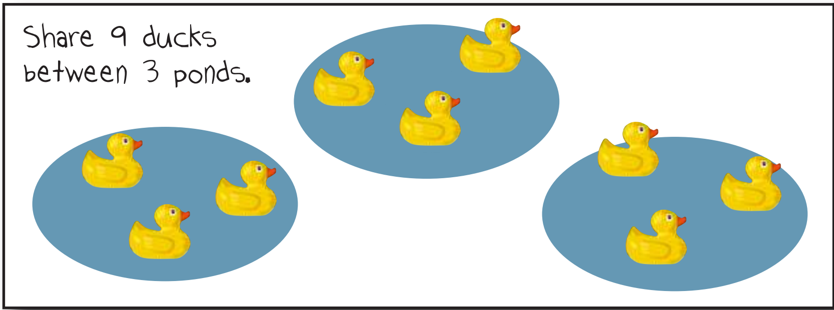
$$\begin{array}{rr} & 80 \\ & 12 \\ & 12 \\ & 1.8 \\ & 0.6 \\ + & 0.09 \\ \hline & 106.49 \\ \hline & 1 \end{array}$$

A = £106.49

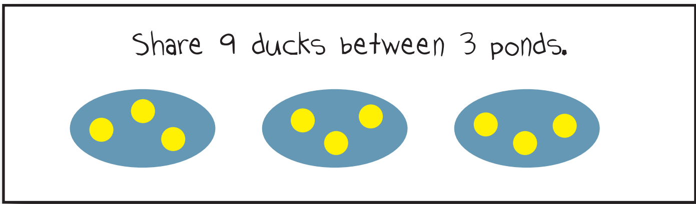
Division

Stage 1 (Foundation Stage - Year 1)

Early division involves sharing equally in practical and real-life contexts.



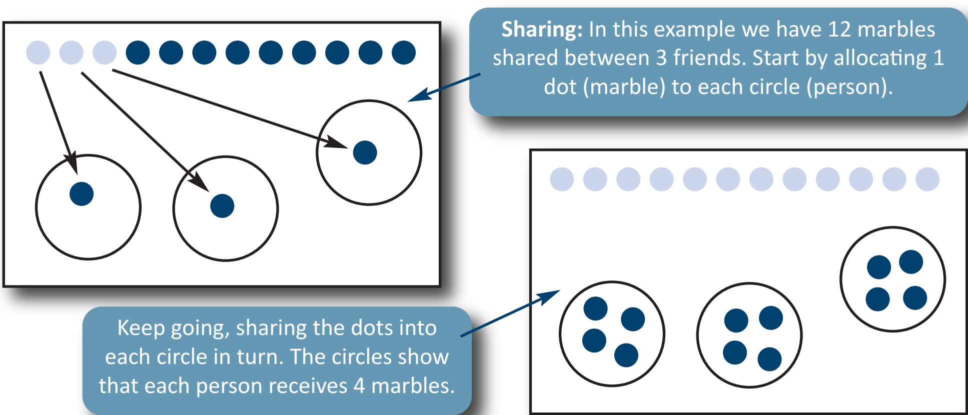
The same problem can be represented with symbols:

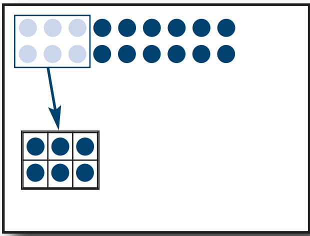


Stage 2 (around Year 2)

At stage 2, children develop their understanding of division as two separate processes:

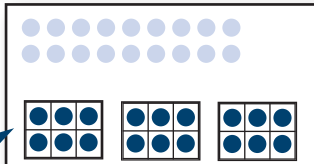
- sharing (eg 12 marbles shared between 3 friends)
- grouping (eg 18 eggs are put into boxes of 6)





Grouping: Here are 18 eggs. This time we are going to put them into boxes (groups) of 6.

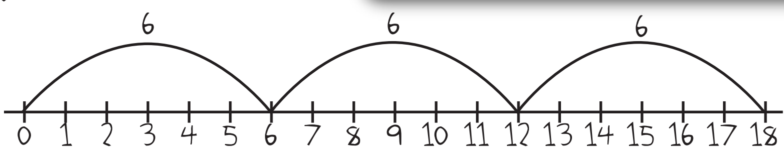
Once we have used all the dots (eggs) we can see how many boxes we need.



In the same way we can use repeated addition to show the same process, that is, we repeatedly add groups of 6 until we can't any longer. It is possible to show this on a numberline.

Note: This is a good example of why we don't put arrows on the jumps on the numberline - the process works equally well starting from 0 and adding as it does starting from 18 and subtracting.

I have 18 eggs which I want to put into boxes of 6.



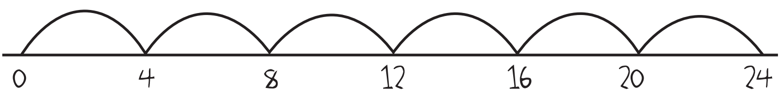
At stage 2 children experience divisions which "work". We deal with the idea of items left over, or **remainders**, at stage 3.

Stage 3 (around Year 3)

Firstly, children can carry out repeated addition on a blank numberline for a calculation with no remainder.

I have 24 pencils in groups of 4. How many lots of 4 are there?

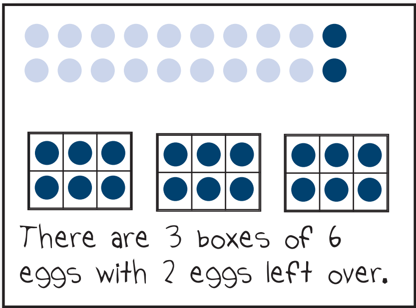
$24 \div 4 = 6$



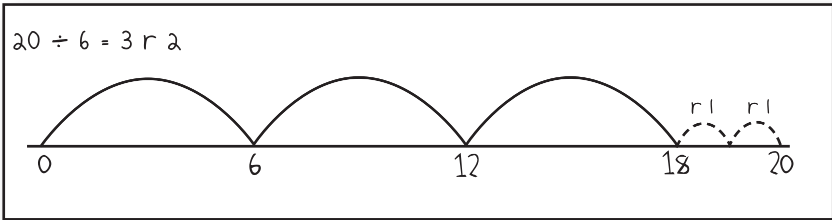
Children will use known multiplication facts to help with division.

I know $4 \times 6 = 24$ so $24 \div 4 = 6$

Then they can see the effect of having a remainder. So, repeating the earlier example of putting eggs in boxes but this time with 20 eggs:

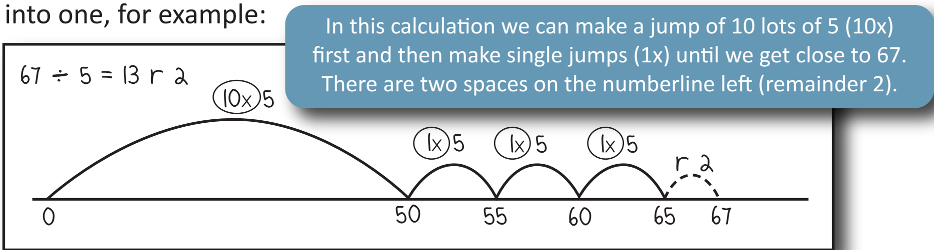


Or on a numberline:

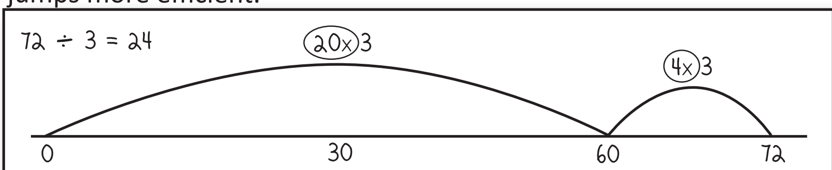


Stage 4 (Year 4)

Stage 4 makes this process more efficient by grouping some of the individual steps into one, for example:



By grouping more than one step as the numbers get larger we can make several larger jumps to the target number. Related multiplication facts are useful to make jumps more efficient.



At this stage, children also divide whole numbers by 10 and 100, extending their knowledge of place value including decimals.

Stage 5 (Year 5)

At this stage children will multiply and divide whole numbers and decimals by 10, 100 and 1000, drawing on known multiplication facts.

Children will continue to use division on a number line for some questions.

Short Division: children will begin to use formal written methods for three-digit (HTU) numbers divided by single-digit (U) numbers.

291 ÷ 3 =
E = 100

$3\overline{)2\ 9\ 1}$

First calculate the number of 3s in 29 - in reality this is the number of threes in 290.

291 ÷ 3 =
E = 100

$\begin{array}{r} 9 \\ 3\overline{)2\ 9\ {}^{2}1} \end{array}$

There are 9 threes, making 27...

...with 2 left over which is carried into the next column as tens.

291 ÷ 3 =
E = 100

$\begin{array}{r} 9\ 7 \\ 3\overline{)2\ 9\ {}^{2}1} \end{array}$

A = 97

Finally, calculate the number of threes in 21.

Children should be able to interpret the remainder as a fraction or decimal, for example:

432 ÷ 5 =
E 400 ÷ 5 = 80

$\begin{array}{r} 8\ 6\ r\ 2 \\ 5\overline{)4\ 3\ {}^{3}2} \end{array}$

A = 86 r 2
= $86\frac{2}{5}$

432 ÷ 5 =
E 400 ÷ 5 = 80

$\begin{array}{r} 8\ 6\ .\ 4 \\ 5\overline{)4\ 3\ {}^{3}2\ .\ {}^{2}0} \end{array}$

A = 86.4

Stage 6 (Year 6)

Finally, long division allows us to tackle calculations where we want to divide by a two-digit number.

Children will continue to use division on a number line for some questions.

$563 \div 24 =$

$E = 600 \div 25 = 24$

$$\begin{array}{r} 2 \\ 24\overline{)563} \\ -\ 480 \\ \hline 83 \end{array}$$

Start by finding the number of 24s in 56 (we know there are no 24s in 5). 2 x 24 is 48, so our first lot is 480. We can therefore put 2 in the tens column at the top (our answer) and subtract our chunk, leaving 83 to do next.

$563 \div 24 =$

$E = 600 \div 25 = 24$

$$\begin{array}{r} 23 \\ 24\overline{)563} \\ -\ 480 \\ \hline 83 \\ -\ 72 \\ \hline 11 \end{array}$$

$A = 23 \text{ r } 11$

Next we look for the number of 24s in 83. As 3 x 24 is 72, we can put these 3 lots of 24 into our answer and again subtract the lot of 72. With 11 remaining, there are no more 24s available.

Children should be able to interpret the remainder as a fraction or decimal, for example:

$432 \div 15 =$

$E = 450 \div 15 = 30$

$$\begin{array}{rl} 28 & \\ 15\overline{)432} & \\ 300 & (15 \times 20) \\ \hline 132 & \\ 120 & (15 \times 8) \\ \hline 12 & \end{array}$$

$\frac{12}{15} = \frac{4}{5}$ $A = 28\frac{4}{5}$

$432 \div 15 =$

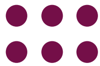
$E = 450 \div 15 = 30$

$$\begin{array}{r} 28.8 \\ 15\overline{)432.0} \\ 30 \\ \hline 132 \\ 120 \\ \hline 12\,0 \\ 12\,0 \\ \hline 0 \end{array}$$

$A = 28.8$

Glossary abc

array - an organised collection of objects, counters or symbols, for example arranged in rows and columns



bridging - the process of using a multiple of 10 or 100 as part of an addition or subtraction calculation, for example 45 + 13 can be thought of as 45 + 5 (50) + 8

decomposition - the standard written method for subtraction (see p10)

difference - the amount by which one number is greater than another - i.e. the result of a subtraction; the difference between 5 and 9 is 4

grid method - a method of calculating multiplication by separating the calculation into sections, each of which easier than the whole (see p12)

least significant digits - the digits with least value - usually the units

number track - a line of numbers used for counting or calculating, each section represents one number

1	2	3	4	5	6	7	8	9	10

numberline - a line where numbers are represented by points on it; numberlines always run from left to right

partitioning - separating a number into its different parts, eg 25 can be partitioned into 20 and 5

remainder - the amount left over in a division which cannot be grouped or shared equally

This booklet was produced by
Girton Glebe Primary School
Cambridge Road, Girton, Cambridge, CB3 0PN
www.girtonglebe.com
(01223) 276484 office@girtonglebe.cambs.sch.uk
If you would like the text of this booklet in a different format or language, please contact the school office.